Status report on modeling the Balbekov square ring in ICOOL

R.C. Fernow
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We are in the process of setting up an ICOOL simulation of Valeri Balbekov's four-sided cooling ring [1] for the MUCOOL experiment, which is shown in Fig. 1. At present we are working with hard-edged models of the ring magnets. The solenoids are modeled with current sheets with magnetic mirrors approximating an iron end clamp. The ultimate goal is to develop a model with more realistic field profiles.

Basic parameters

Starting with the eight 45^o combined function bending magnets we take [1]

B_D = 1.453 T
ρ = 0.52 m
n = 1/2 (field index)

This fixes the central momentum

p_o = 0.226511 GeV/c

Then the bend angle of 45^o fixes the arclength in the dipole

s_D = 0.408407 m

We take the circumference of the ring from the paper

C = 36.955 m

With an average momentum p_o going around the ring and assuming the particles are muons, the revolution time is

T_{REV} = 136.020 ns

For the stated RF frequency of 201.25 MHz, the RF period is 4.968 ns and the harmonic number is 27.37, not the integer value 28 given in the paper. To fix this we decided to use the harmonic number

h = 28

and readjust the RF frequency accordingly

f_{RF} = 205.8524 MHz

The corresponding RF period is 4.858 ns.

Reference particle

ICOOL uses an internal reference particle to set the absolute phases of the RF cavities. The algorithm chosen starts with an on-axis particle with momentum p_o and all stochastic processes turned off. The particle is tracked thru all regions except RF cavities. We took the length of liquid hydrogen absorber from the paper

$L_{ABS} = 1.33$ m

The particles loses 44.5 MeV in the absorber. This energy is made up in the 16 RF cavities in the straight section. The cavities are assumed to have pillbox fields and to have a length and peak on-axis gradient

$L_{CAV} = 0.32$ m
G = 15 MV/m

The phase algorithm assumes the reference particle gains energy at a constant rate while crossing each cavity. The parameter that specifies this rate

GRADREF = 7.865 MV/m

was adjusted so that the reference particle had momentum p_o after leaving the 16th cavity. This fixes the reference particle trajectory. The time that the reference particle crosses the center of each cavity is determined and used to set the time that cavity is at zero-crossing of the electric field. With the appropriate parameters set to ~9 significant figures and using the double precision version of the code, the reference particle stayed on-axis for 15 complete turns without using the solenoid focusing. The final radial position error was ~0.2 μm and the transverse momentum error was ~5 10^{-8} GeV/c.

Real on-axis, on-momentum particle

Now that the absolute cavity phases have been fixed we can look at tracking real particles. We assume that the cavities in the other three quadrants of the ring have the same absolute phase as the corresponding cavity in the first quadrant. We take a particle with momentum p_o. In order for the particle to get accelerated in the cavities we shift its launch time relative to the reference particle. By setting a parameter

T0REF = -0.472 ns

the particle phase crossing the cavities is just right to end up with momentum p_o after leaving the 16th RF cavity. This corresponds to a synchronous phase of 35.0^o.

However, there is a time shift of 8.8 ps between the real particle and the reference particle when leaving the 16th cavity. This means the real particle will enter the first cavity in the next quadrant at a different phase than it entered the first cavity in the first quadrant. Part of this time difference comes about because the real particle does not have the same momentum profile thru the cavities and absorbers that was assumed for the reference particle. We attempted to reduce this error by setting an additional phase offset in each cavity in the first quadrant so that the momentum profile matched that of the reference particle. These offsets were small, the largest being 0.27^o. However, the resultant time shift after the 16th cavity was only reduced to 7.8 ps with this procedure.

We showed that this time difference is small enough for multiturn tracking to work. A special run with the dipole fields set to 0 and no transverse focusing showed that the track successfully traversed a distance equivalent to 15 times the circumference, which in turn shows that this is a phase-stable solution.

With the dipole fields on and transverse focusing on the particle completed 15 turns around the ring. At the end the particle was 6 mm off the axis and had a transverse momentum of 9 MeV/c.

Bend region

We checked the linear dispersion suppression in the system by looking at a simplified system consisting of the dipole-short solenoid-dipole combination. The wedge at the center of the solenoid was removed. An on-axis particle was launched with p = 0.220 GeV/c. We adjusted the overall current density scale factor in order to minimize the transverse beam position and momentum at the end of the system. This produced a peak field of 2.67 T and a field at the solenoid hard edge of 2.025 T, in rough agreement with Fig. 3 in [1]. After adjustment the final radial position was 2 μm and the transverse momentum was 0.2 keV/c. Adjusting the two coil current densities in the field flipping short solenoid independently did not improve the results.

We also used the dipole-short solenoid-dipole combination to check for proper operation of the LiH wedge. An on-axis beam was used with momentum spread σ_{PZ} = 20 MeV/c. The beam is dispersed in x by the first dipole. The solenoidal field between the dipole and the wedge rotates the dispersion so that it lies mainly in the y direction going into the wedge. The wedge angle was [1]

$$\alpha_W = 25.4^o$$

The wedge is oriented vertically with the apex touching the beam axis. The effect of the wedge on the kinetic energy [MeV] of the beam is given in Table 1.

	y > 0	y < 0
Before the wedge	129.9 ± 10.9	158.9 ± 11.1
After the wedge	129.9 ± 10.9	156.5 ± 9.4

We see that, as desired, the average energy and the energy spread are reduced in one half of the vertical dimension and left alone in the other half.

The dispersion functions following the second dipole are shown in Figs. 2 and 3. By design the dispersions vanish exactly at KE = 138 MeV (p = 220 MeV/c). The momentum region over which the displacement vanishes is small, particularly in x.

An indication that the motion thru the wedges is not properly synchronized yet in the model can be seen in Fig. 4, which shows p_Z for a particle that starts on-axis with p = 220 MeV/c. Although the particle is tracked thru 15 complete turns without being lost, the wedges do not damp the longitudinal momentum spread.

Initial beam conditions

We assume the initial beam parameters were [1]

$\sigma_X = \sigma_Y$ = 4 cm
$\sigma_{PX} = \sigma_{PY}$ = 32 MeV/c
σ_Z = 8.9 cm
σ_{PZ} = 18 MeV/c (without correlation)

The initial beam was given a transverse amplitude-momentum correlation according to the prescription in [1]. Let us define the Balbekov amplitude

$$A_B = \sqrt{\left(\frac{r}{\beta_\perp}\right)^2 + \left(\frac{p_\perp}{m_\mu c}\right)^2}$$

The quantities r and $p_\perp$ are randomly chosen to determine the initial value of this amplitude. The quantity $\beta_\perp$ is fixed at 30 cm, as in [1]. Once A_B is known, we set the total energy of the particle according to

$$E = E_{REF}\sqrt{1 + A_B^{\,2}} + \Delta E$$

The quantity E_{REF} is fixed at 250 MeV, as in [1], and ΔE was selected randomly. Fig. 5 shows the resulting correlation of longitudinal momentum versus the Balbekov amplitude when ΔE=0. It is interesting to compare this with the related Palmer amplitude

$$A_P = \sqrt{\left(\frac{r}{\beta_\perp}\right)^2 + (x')^2 + (y')^2}$$

Fig. 6 shows the same beam distribution that was prepared with the Balbekov amplitude plotted as a function of the Palmer amplitude. Beams prepared with the two amplitudes are clearly not equivalent.

Full beam simulations

We are just beginning to look at full beam simulations including scattering and straggling, but no decays. The first attempt lost all the beam after ~310 m. The major problem seems to be particles falling out of the RF bucket. Fig. 7 shows the longitudinal phase space after one turn. The beam is located at the entrance to the long straight section. The transmission after one turn was 56%.

Notes and references

[1] V. Balbekov et al, Muon ring cooler for the MUCOOL experiment, Proc. PAC 2001.

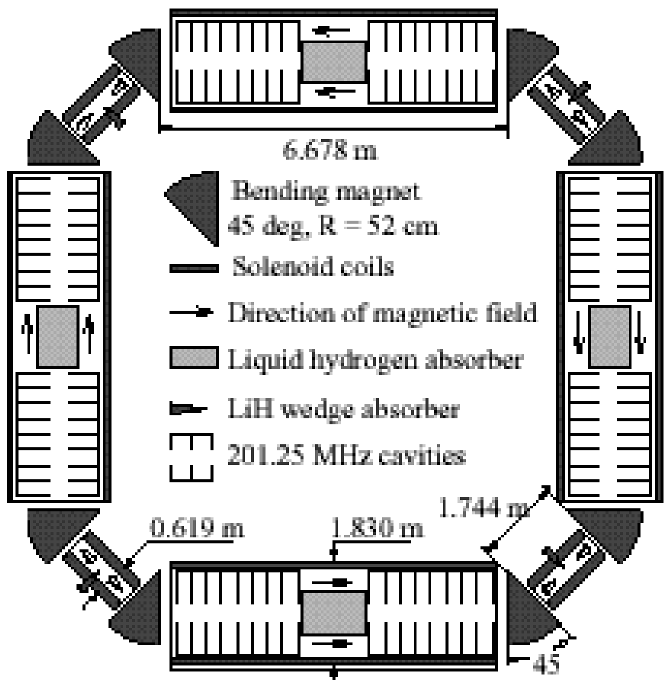

Figure 1: Layout of the ring cooler.

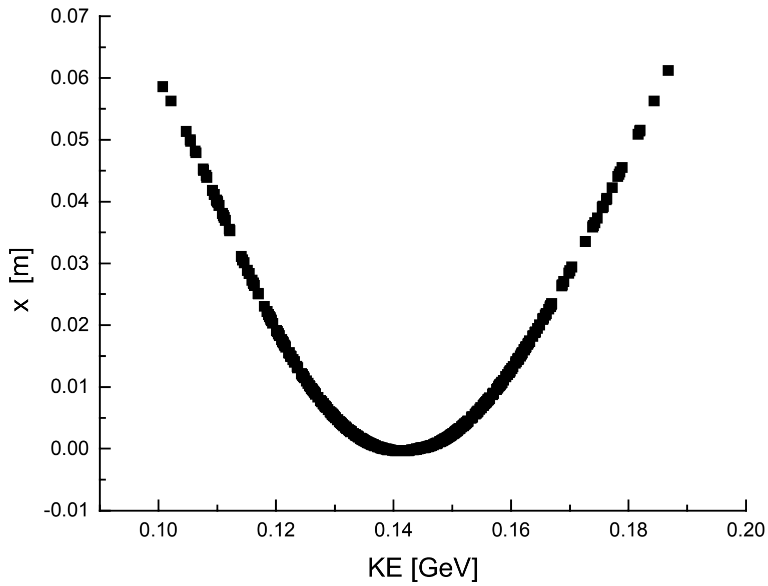

Figure 2 Horizontal dispersion after the second dipole.

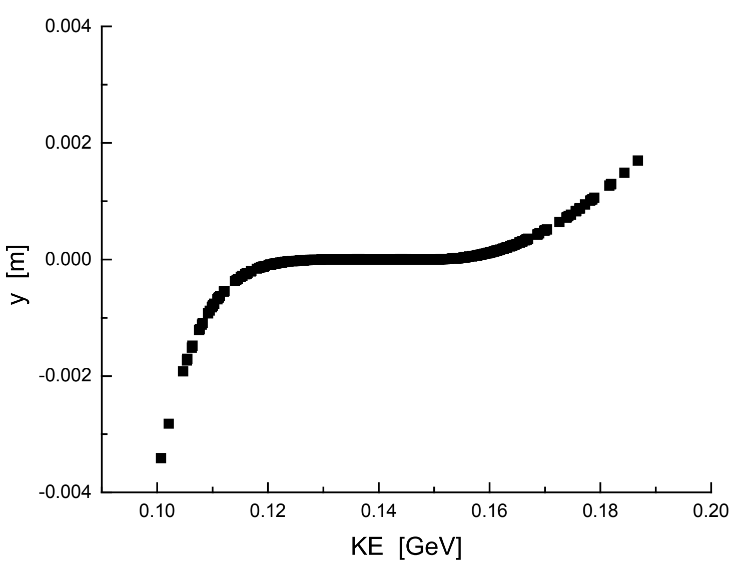

Figure 3 Vertical dispersion after the second dipole.

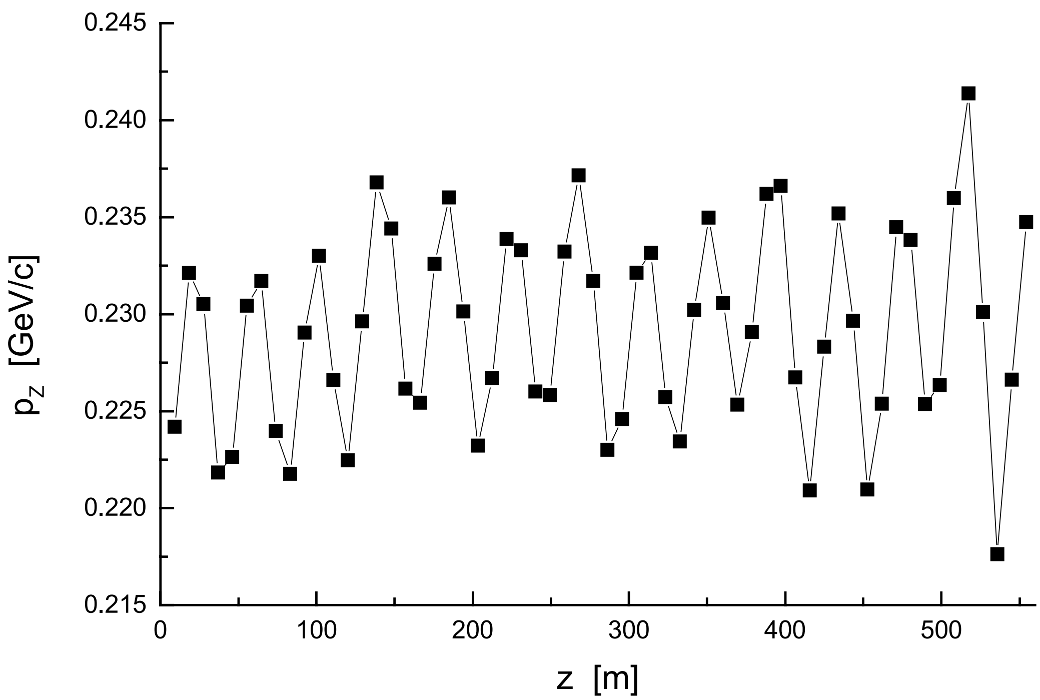

Figure 4 Longitudinal momentum for a track starting on-axis with p = 220 MeV/c.

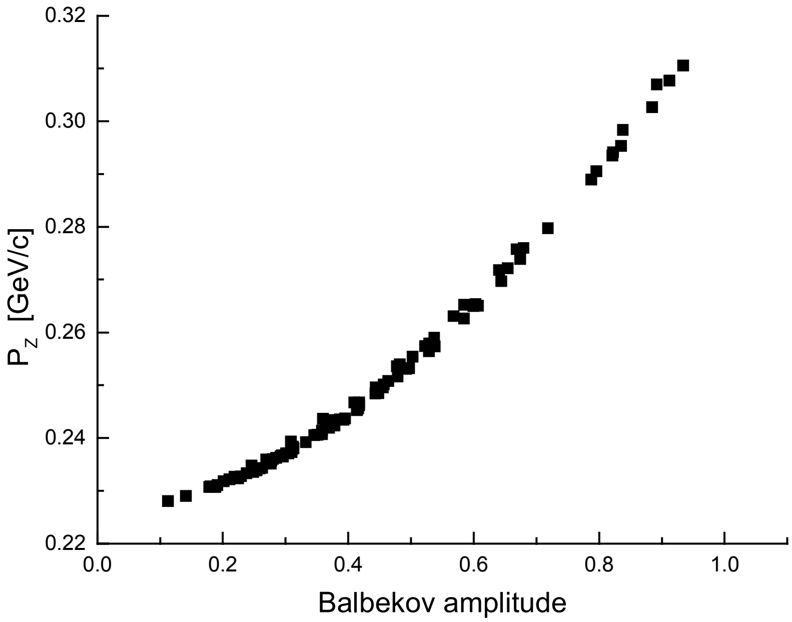

Figure 5 Initial momentum correlation as a function of the Balbekov amplitude

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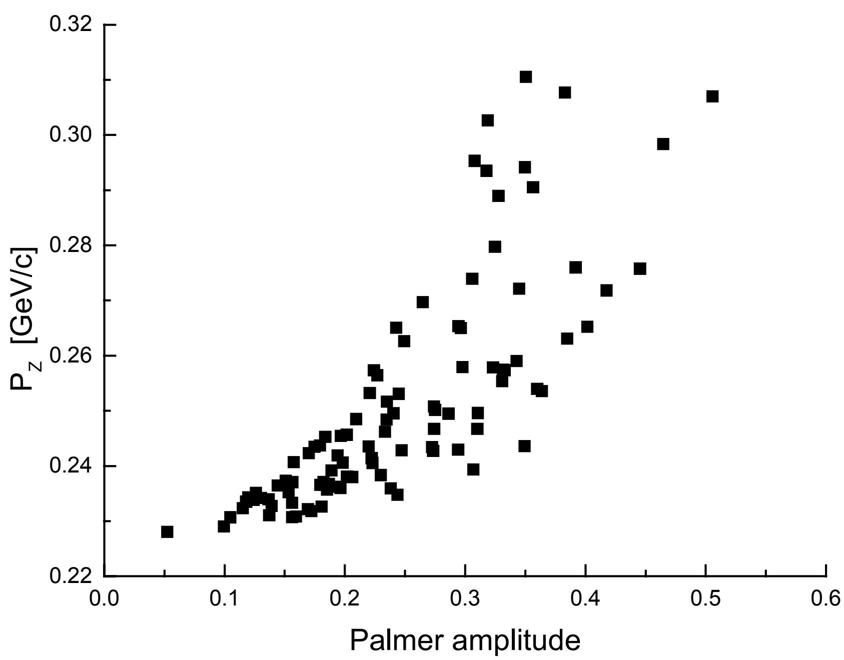

Figure 6 Initial momentum correlation of a beam prepared with the Balbekov correlation as a function of the Palmer amplitude.

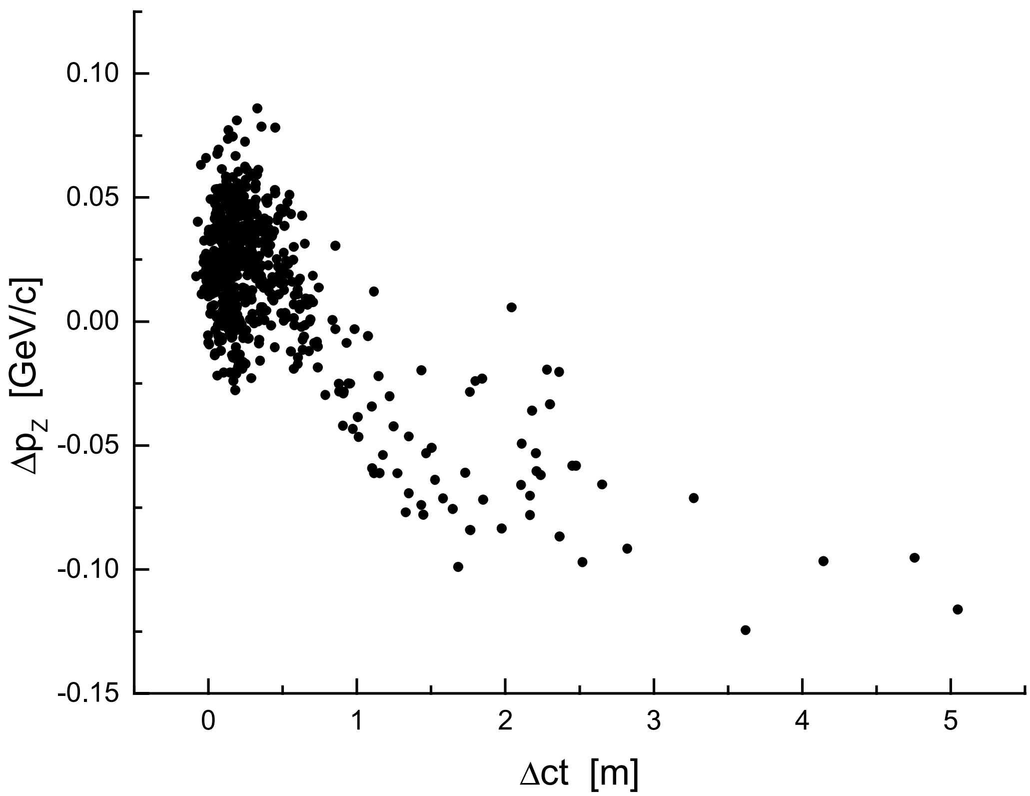

Figure 7 Longitudinal phase space after one turn.